Mathletics

Series D Student

Patterns and Algebra

My name

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Series D – Patterns and Algebra

Contents

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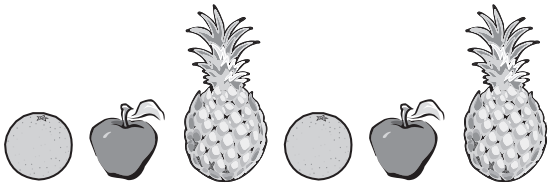
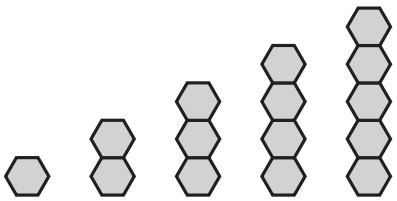
Patterns and functions – identifying and creating patterns

Look around you, can you see a pattern? A pattern is an arrangement of shapes, numbers or colours formed according to a rule. Patterns are everywhere, you can find them in nature, art, music and even in dance! You can make a pattern out of anything. Patterns can grow or repeat.

Here is a pattern made out of fruit that repeats:



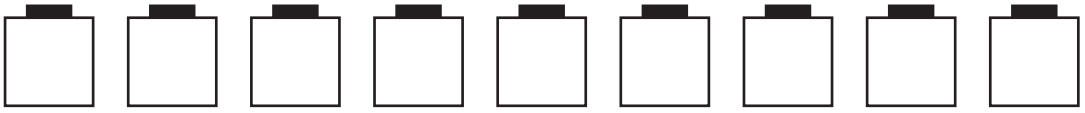
Here is a pattern made out of hexagons that grows:



1 **Look at this colour pattern made with cubes. What comes next? Write the letters on the blank cubes then colour them in.**

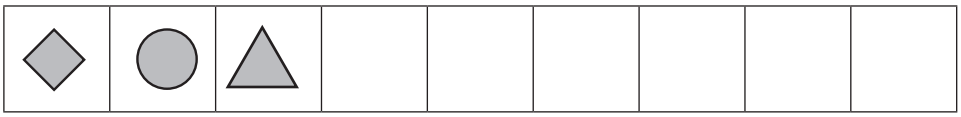
B – Blue
R – Red
G – Green
Y – Yellow

2 **Make your own colour pattern with these cubes using colours from the box above. You can colour them or just write the letter.**

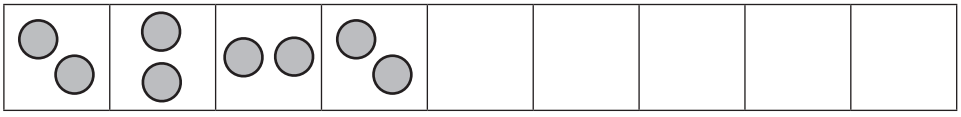


3 **In these shape patterns, draw the missing shapes.**

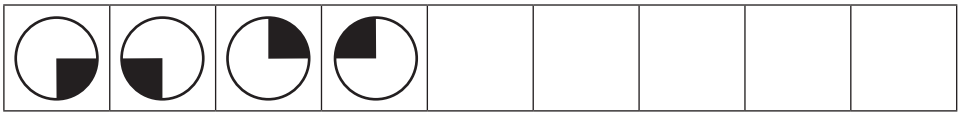
a



b



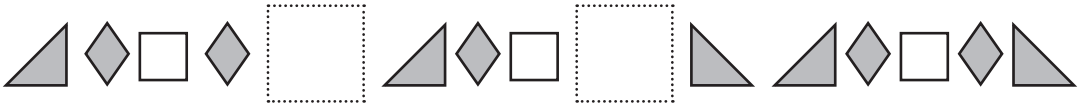
c



To work out what comes next, look out for the sequence of shapes that make up the rule.

THINK

4 **Complete the shape patterns by drawing 2 missing shapes on each line:**

a 

b 

5 **Look at the repeating letter pattern and write in the missing letters. You will see that each pattern is a word repeated.**

a B I C ___ C ___ E B I ___ Y ___ L E B ___ C Y C L ___

b C ___ L O ___ R S C O ___ O U R ___ C ___ L ___ U R S

6 **Follow the directions to create 2 growing patterns:**

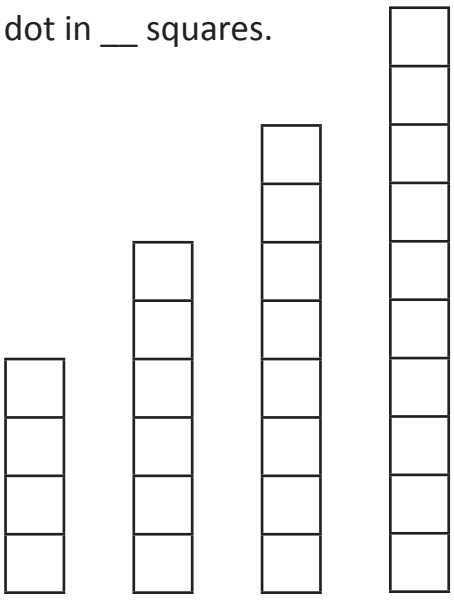
a

Tick 2 squares and put a dot in 2 squares.

Tick 3 squares and put a dot in 3 squares.

Tick 4 squares and put a dot in 4 squares.

Tick __ squares and put a dot in __ squares.



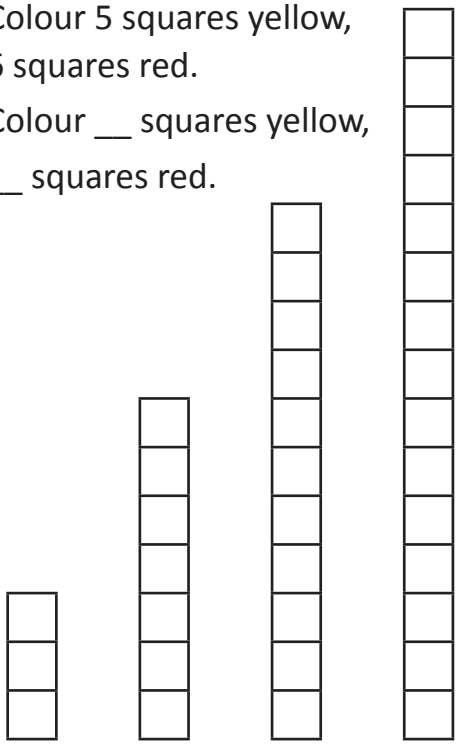
b

Colour 1 square yellow, 2 squares red.

Colour 3 squares yellow, 4 squares red.

Colour 5 squares yellow, 6 squares red.

Colour __ squares yellow, __ squares red.



Patterns and functions – skip counting

Skip counting is a good skill to have because you can see number patterns more easily which makes you better at maths. You can also count things much faster!

This is a skip counting pattern of 2 on a hundred grid.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1 **Colour the skip counting pattern on each hundred grid:**

a Show the 5s pattern.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

b Show the 10s pattern.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

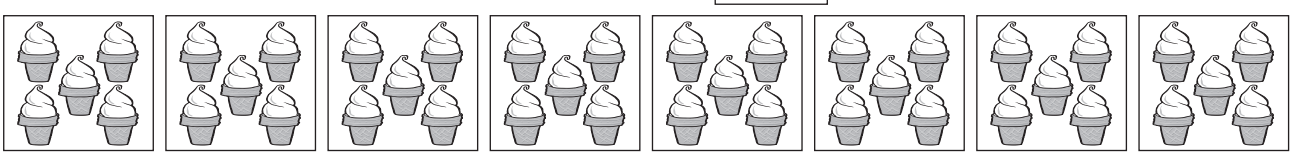
c What do you notice?

_____

2 **Complete these skip counting patterns:**

a	60	65	70			85		95
b	17	22	27		37		47	
c	100	95			80		70	
d	102	92			62			

3 **Count the ice creams. How many are there?** ☐



Patterns and functions – skip counting

4 **Colour the skip counting pattern on each hundred grid:**

a Show the 3s pattern.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

b Show the 4s pattern.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

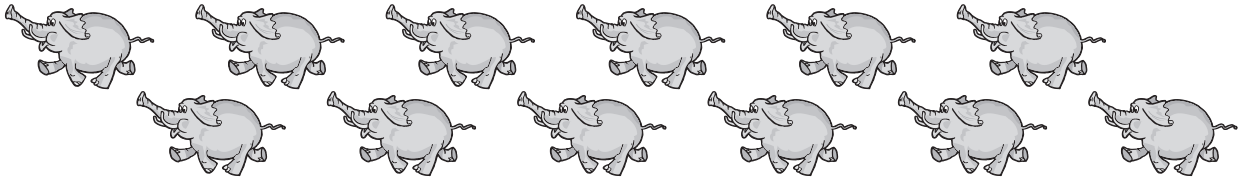
5 **Complete the missing numbers in these skip counting patterns:**

a	36			27	24			
b	12		20	24			36	40
c	50		46	44			38	
d	27			57		77	87	

6 **How many objects altogether? Use skip counting.**

a How many candles? ☐

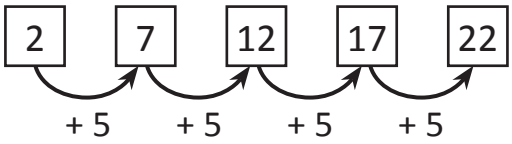
b How many legs? ☐



Patterns and functions – completing and describing patterns

Skip counting in the hundred grid starting at zero, is a good way to begin looking at number patterns. Now let's look at number patterns that start at numbers bigger than zero.

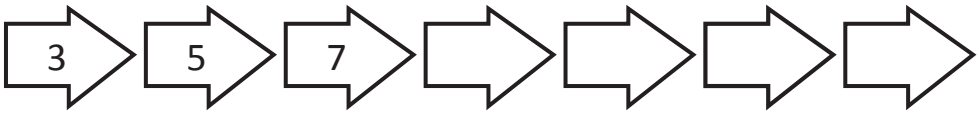
This pattern starts at 2.



The rule is: Add 5.

1 **Complete the missing numbers in each pattern:**

a Rule: Add 2



b Rule: Add 4

c Rule: Subtract 5

2 **Continue the pattern from the starting number:**

a	Add 10	11						
b	Add 5	55						
c	Subtract 4	40						

3 **Finish each pattern and write the rule:**

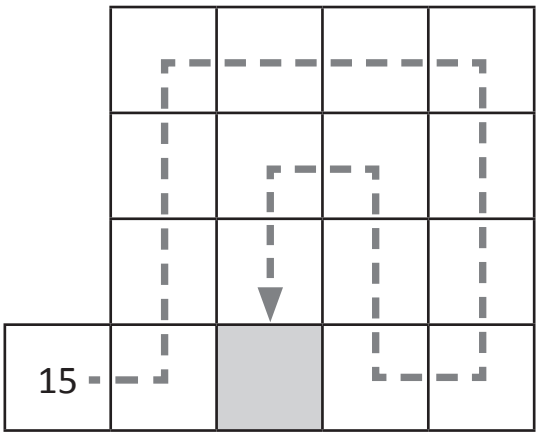
a 5, 8, 11, ___, ___, ___, ___, ___ Rule: ____

b 7, 14, 21, ___, ___, ___, ___, ___ Rule: ____

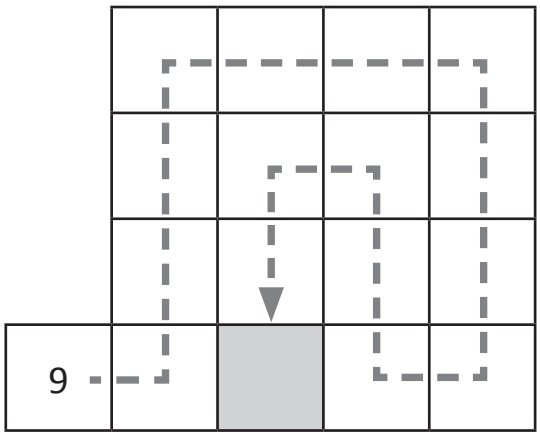
c 72, 63, 54, ___, ___, ___, ___, ___ Rule: ____

4 **Fill these snail grids with these patterns. You can use a calculator.**

a Skip count by 15:



b Skip count by 9:



5 **Check these patterns with a calculator. They all have mistakes in them. Find the mistakes, circle them and write the corrections underneath.**

a

12	50	88	126	164	204	242	280

b

84	77	70	63	56	50	43	36

c

17	59	101	143	185	229	271	313

6 **Roll a set of dice to make a 2 digit number. This is the starting number. Write it in the first space. Then continue the sequence by following the rule.**

a Rule: + 10

b Rule: + 3

c Rule: + 4

Patterns and functions – number patterns in tables

When we use number patterns in tables it can help us to predict what comes next. Look at the table below. Once we work out how the pattern works, we can predict the total number of feet for any amount of students.

This table shows us that when there is 1 child there are 2 feet.

When there are 2 children there are 4 feet and so on.

We can see that the rule for the pattern is to multiply the top row by 2 to get the bottom row each time.

Number of children	1	2	3	4	5	20
Number of feet	2	4	6	8	10	40

× 2

To find out how many feet 20 children would have, we don't need to extend the table, we can just apply the rule.

1 **Try these number pattern tables.**

At a party, one child receives 3 chocolates. Complete the table to show how many chocolates different numbers of students receive. Show how many 20 receive.

Number of children	1	2	3	4	5	20
Number of chocolates	3					

2 **Alfred is a type of alien from the Planet Trampolon. The surface of Planet Trampolon is like walking on a trampoline. That is why Alfred and all his race of aliens need 3 legs – for extra balance. They also have 2 antennae and 4 fingers on each hand.**

Complete the number pattern tables to show the number of different body parts for different amounts of aliens.

a

Number of aliens	1	2	3	4	20
Number of antennae	2				

b

Number of aliens	1	2	3	4	20
Number of fingers on each hand	4				

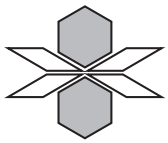
c

Number of aliens	1	2	3	4	20
Number of legs	3				

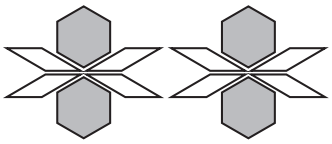
Patterns and functions – growing shape patterns

Let's look at this growing pattern:

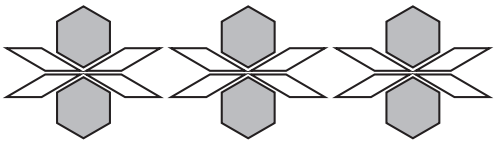
1 butterfly uses 2 hexagons.



2 butterflies use 4 hexagons.



3 butterflies use 6 hexagons.



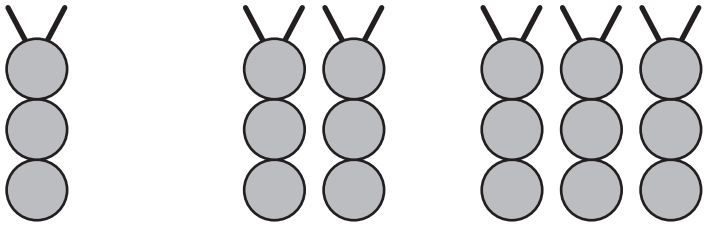
How many hexagons would 10 butterflies use?

There is a way we can do this without using pattern blocks.

We just look for a pattern. The pattern is that you need to double the amount of hexagons for each butterfly. So for 10 butterflies, you would need 20 hexagons.

1 **Here are some pictures made from shapes.**

a Fill in the blanks for each part of the pattern and draw what comes next:



1 ant uses 3 circles.

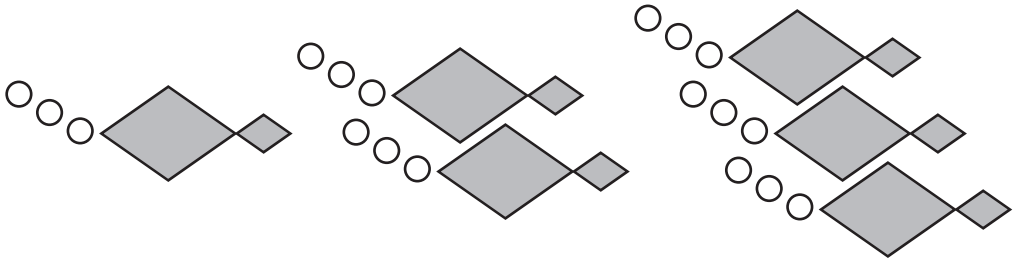
2 ants use _____ circles.

3 ants use _____ circles.

_____ ants use _____ circles.

b How many circles would you use for 10 ants? _____

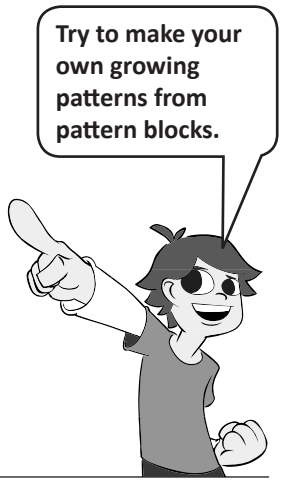
c The first fish is made up of 5 shapes. Fill in the boxes for 2 fish and 3 fish:



1 fish uses 5 shapes.

2 fish use _____ shapes.

3 fish use _____ shapes.

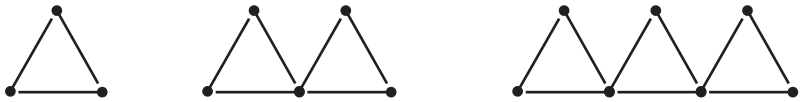


d How many shapes would you use for 10 fish? _____

Patterns and functions – matchstick patterns

Number patterns in tables can help us with problems like this. Mia is making this sequence of shapes with matchsticks. How can she find out how many she needs for 10 shapes?

Shape 1 **Shape 2** **Shape 3**



Shape number	1	2	3	4	5	10
Number of matchsticks	3	6	9	12	15	30

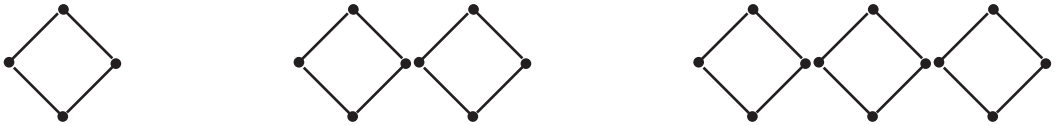
× 3

To find out how many matchsticks are needed for 10 triangles, we don't need to extend the table, we can just apply the function rule:

Number of matchsticks = Shape number × 3

1 **Complete the table for each sequence of matchstick shapes and find the number of matchsticks needed for the 10th shape.**

a Shape 1 Shape 2 Shape 3



Shape number	1	2	3	4	5	**10**
Number of matchsticks	4					

b Shape 1 Shape 2 Shape 3

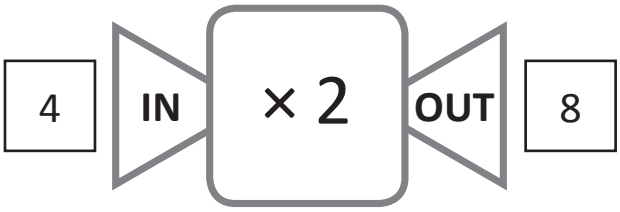
Shape number	1	2	3	4	5	**10**
Number of matchsticks	5					

c Draw the fourth shape in the sequence above:

Patterns and functions – function machines

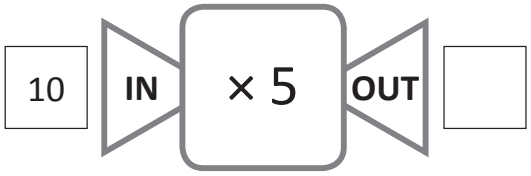
This is a function machine.

Numbers go in, have the rule applied, and come out again.

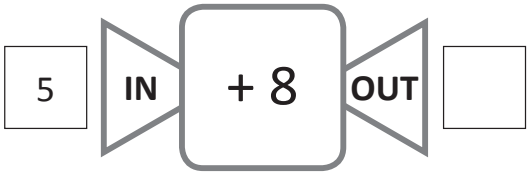


1 **What number will come out of these function machines?**

a

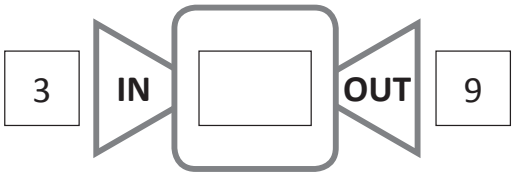


b

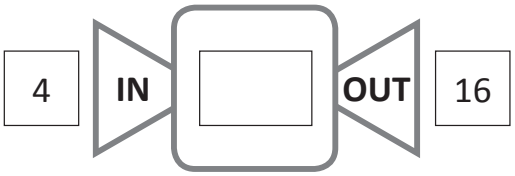


2 **Write the rule on these function machines:**

a

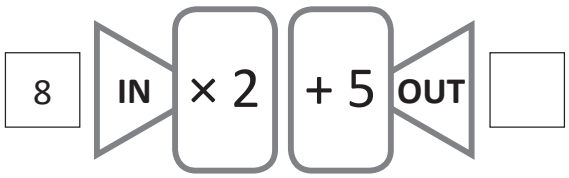


b

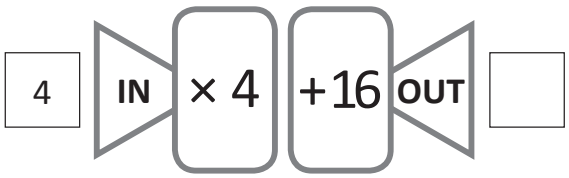


3 **What number will come out of these double function machines?**

a

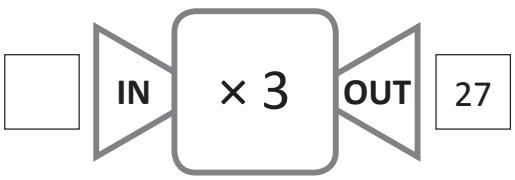


b

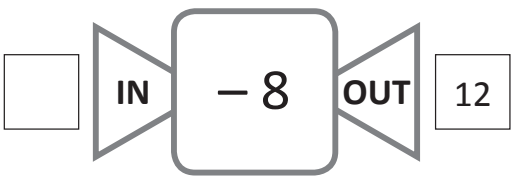


4 **Write the number that went into these function machines:**

a



b



Harry and Tortista

solve

Getting ready

Read the problem below and use your knowledge of number patterns to solve the problem.

What to do

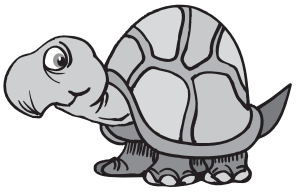
Harry and Tortista constantly argued over who was the faster runner out of the pair. To settle the dispute once and for all, they decided to race each other. Harry was so confident that he could beat Tortista, he gave Tortista a head start of 3 km.

If Harry runs 1 km every 3 minutes and Tortista runs 1 km every 4 minutes, who will win the 12 km race?

Complete the table for Harry and Tortista to find out:

Harry	
km	**mins**
0	0
1	3
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

Tortista	
km	**mins**
3	0
4	4
5	
6	
7	
8	
9	
10	
11	
12	



Rows and columns — apply

Getting ready

This is a game for 2 players. You will need 3 dice, this page and 12 counters each in 2 different colours.

What to do

Player 1 rolls all 3 dice, adds them together and puts this value in the first function rule. *For example, if they roll a 3, 5 and 2, they should add these and get 10. They put 10 into the first rule and get 10 + 5 = 15. Player 1 places one of their counters on 15. Then Player 2 repeats these steps.*

Keep taking turns using a different function rule each time. If the answer is already taken, you lose a turn.

The winner is the first person to get rid of all their counters.

Function Rule 1	**Function Rule 2**	**Function Rule 3**
◈ + 5	2 × ⊙	▣ − 2

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	1	2	3	4
5	6	7	8	9	10	11	12	13	14
15	16	17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32	33	34
35	36	1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27	28

What to do next

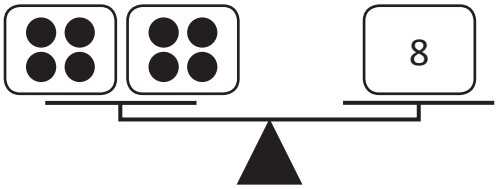
Change the object of the game. For example, the winner might be the person who has their counters on the most even numbers.

Equations and equivalence – introducing equations

Look at these balanced scales.

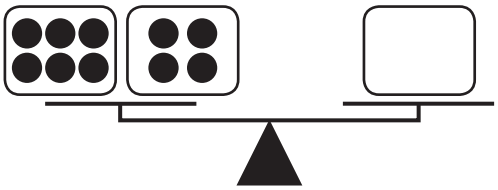
In each box on the left there are 4 dots and on the other side is the number 8.

This makes sense because it shows the equation 4 + 4 = 8. An equation is a sum with an equals symbol. One side must equal or balance the other just like these scales.



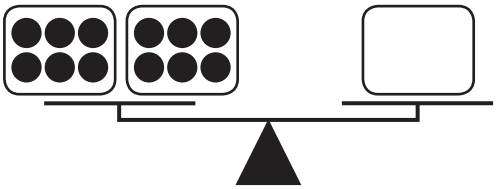
1 **Balance each set of scales by writing a number in the box. Then write the matching equation:**

a



☐ + ☐ = ☐

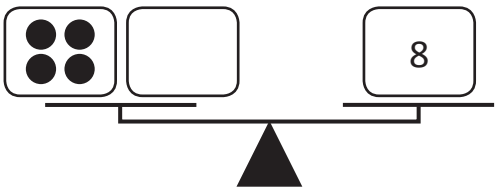
b



☐ + ☐ = ☐

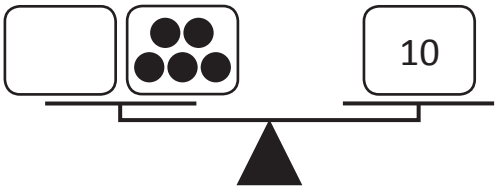
2 **Again, balance each set of scales but this time add the missing dots to the empty box:**

a



☐ + ☐ = ☐

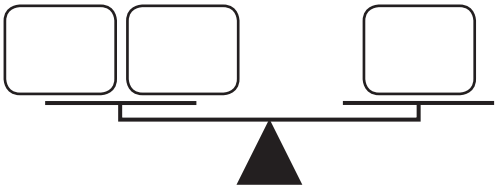
b



☐ + ☐ = ☐

3 **This time, create your own equation and show it on the balanced scales:**

a

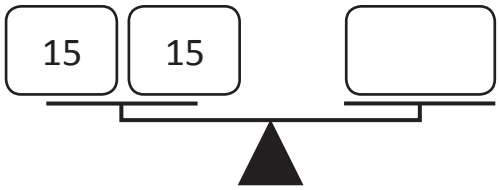


☐ + ☐ = ☐

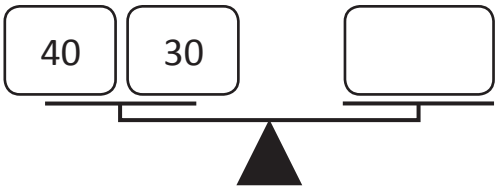
Equations and equivalence – introducing equations

4 **Balance each set of scales by writing the missing number in the box.**

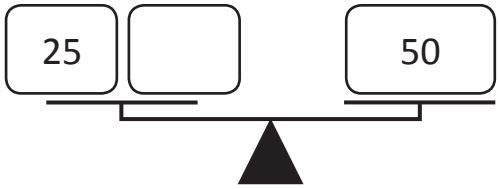
a



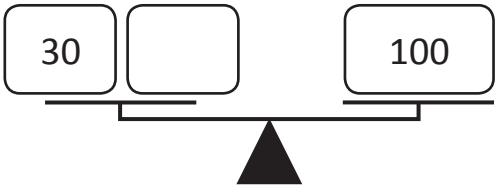
b



c

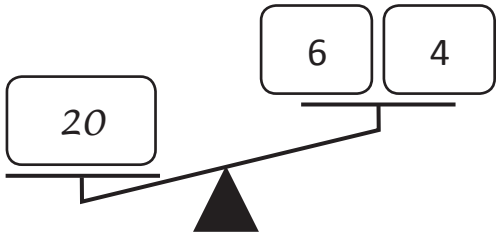


d

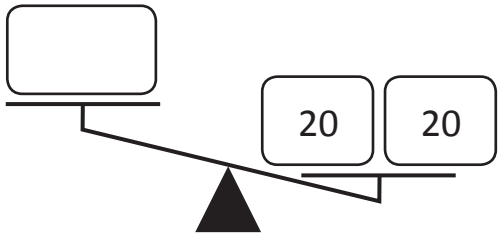


5 **These scales are not balanced. This shows that the equation is not equal. One side is greater than the other. Write a number in the box to make these true. The first one has been done for you.**

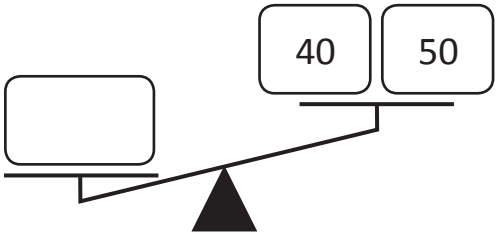
a



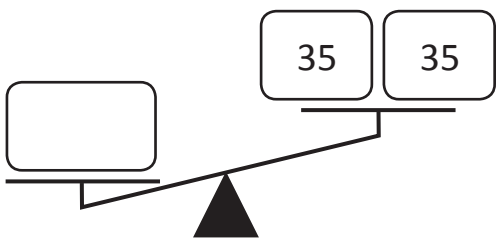
b



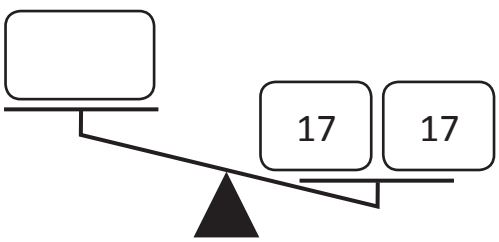
c



d

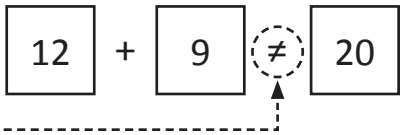


e



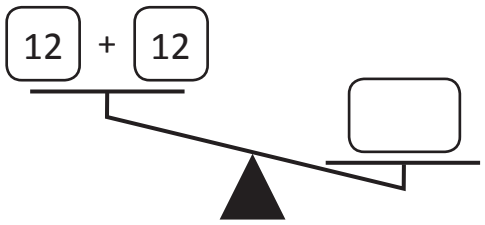
Equations and equivalence – not equal to symbol

When two sides of an equation are not balanced, it means that they are not equal. To show that an equation is not equal, we use the not equals symbol like this:



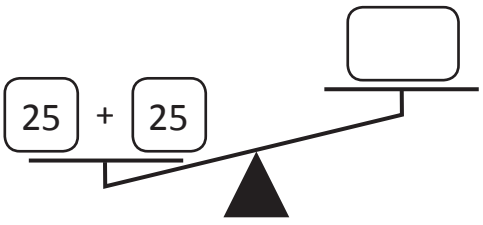
1 **Balance each set of scales by writing a number in the box. Then write the matching equation.**

a



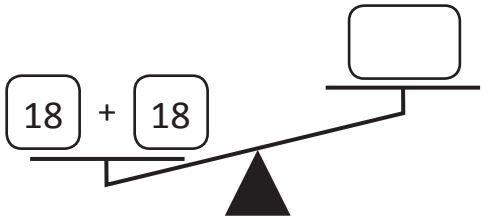
12 + 12 ≠ ☐

b



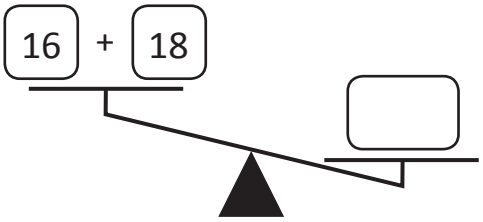
☐ + ☐ ≠ ☐

c



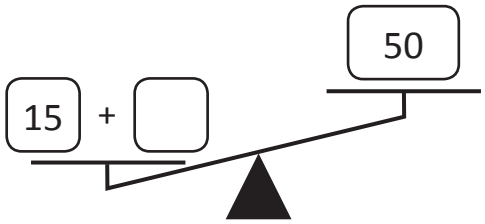
☐ + ☐ ≠ ☐

d



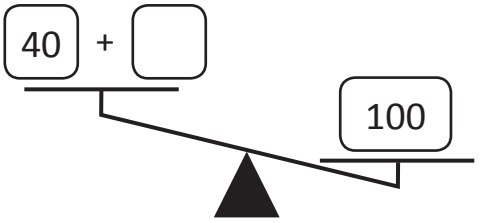
☐ + ☐ ≠ ☐

e



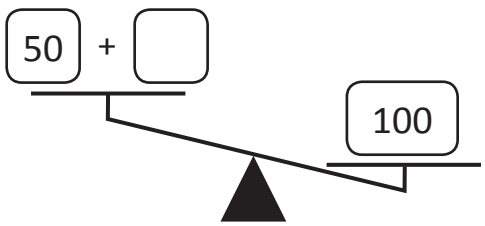
☐ + ☐ ≠ ☐

f



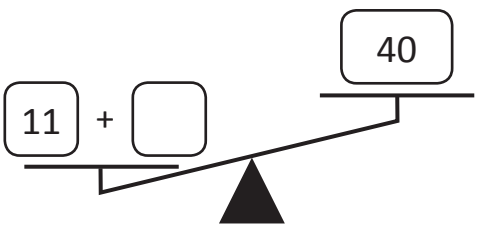
☐ + ☐ ≠ ☐

g



☐ + ☐ ≠ ☐

h



☐ + ☐ ≠ ☐

Equations and equivalence – not equal to symbol

2 **Practise using the *equals to* (=) or *not equals to* (≠) symbol in these problems. Roll 2 dice and write the number in each box. Then, make the equation true by either writing = or ≠ in the circle.**

a ☐ + ☐ ◯ 12

b ☐ + ☐ ◯ 6

c ☐ + ☐ ◯ 8

d ☐ + ☐ ◯ 12

e ☐ + ☐ ◯ 10

f ☐ + ☐ ◯ 7

3 **Complete the equations below only using the numbers in the cards. Look carefully to see whether it is = or ≠ .**

16	20	4	10	7	3

a ☐ + ☐ = ☐

b ☐ + ☐ ≠ ☐

c ☐ + ☐ = ☐

d ☐ + ☐ ≠ ☐

4 **Roll a die and write the number in any star that balances the equation. Your aim is to balance as many equations as you can out of 6 rolls of the die. For numbers that do not balance the equations, use an ≠ symbol.**

a 6 + ☆ ☐ 10

b 5 + ☆ ☐ 9

c 9 + ☆ ☐ 12

d 11 + ☆ ☐ 15

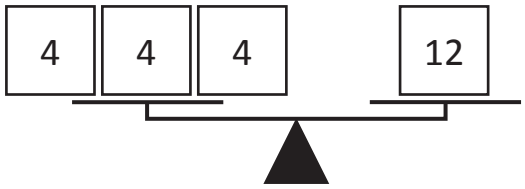
e 3 + ☆ ☐ 6

f 4 + ☆ ☐ 8

g How did you go? _____

Equations and equivalence – balanced equations using + and ×

There are 2 different equations we could write for one set of balanced scales.

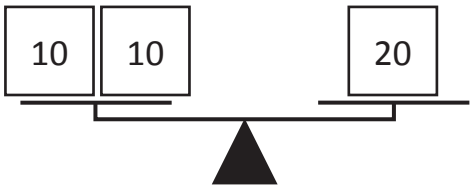


4 + 4 + 4 = 12

3 × 4 = 12

1 **Work out the values of the symbols in each problem.**

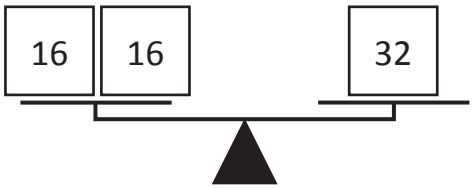
a



□ + □ = 20

2 × □ = 20

b

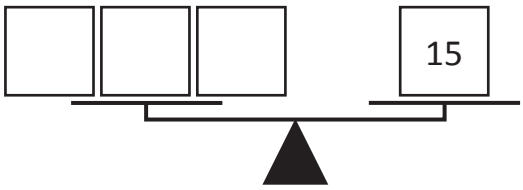


□ + □ = 32

2 × □ = 32

2 **This time work out which number should go in the symbol.**

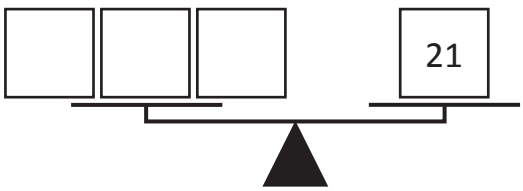
a



□ + □ + □ = 15

□ × 5 = 15

b

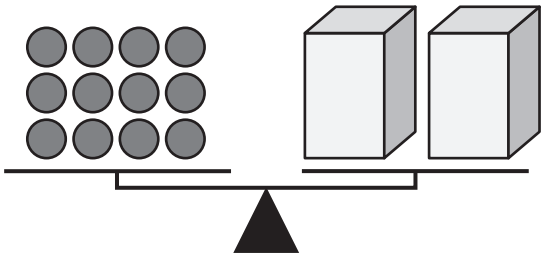


□ + □ + □ = 21

□ × 7 = 21

Equations and equivalence – balanced equations using + and ×

How many dots are inside each box? On one side there are 12 dots and on the other side, there are 2 boxes. Because the equation is balanced, there must be 6 in each box.

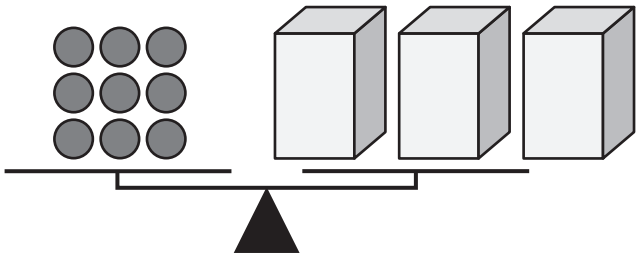


There are 2 different equations we could write for one set of balanced scales.

$6 + 6 = 12$

$2 \times 6 = 12$

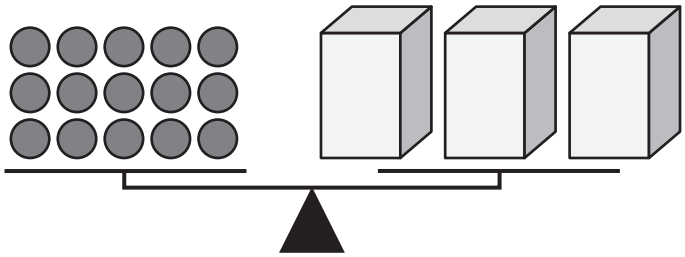
3 **How many dots are inside each box?**



$\square + \square + \square = 9$

$3 \times \square = 9$

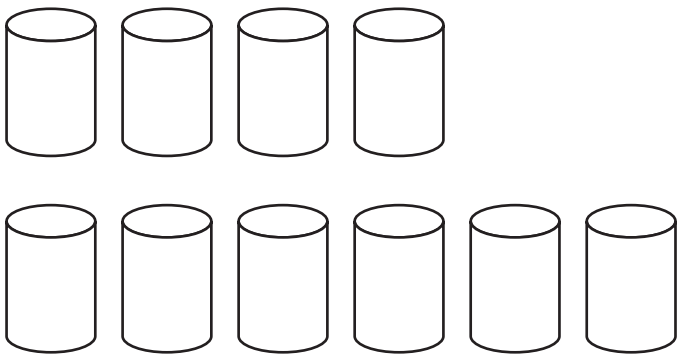
4 **How many dots are inside each box?**



$\square + \square + \square = 15$

$3 \times \square = 15$

5 **If there are 16 dots in these 4 cylinders, how many dots are there in 6 cylinders? Show your working.**



$4 \times \square = 16$

$\square \times \square = \square$

Equations and equivalence – writing equations for word problems

We can use symbols to stand for the unknown number in word problems. Read this word problem.

Jess and Jo went on an Easter egg hunt. Jess found 3 eggs and Jo found 7 eggs. How many did they find altogether?

The equation for this problem is: 3 + 7 = ☆

☆ = 10

Now read this problem:

Jess and Jo went on an Easter egg hunt. If 10 eggs were found altogether and Jo found 7 eggs, how many did Jess find?

The equation for this problem is: 7 + ☆ = 10

☆ = 3

1 **Warm up with these. Find the value of the symbols in each equation.**

a 3 × ☺ = 9

☺ = ☐

b 9 × ✦ = 36

✦ = ☐

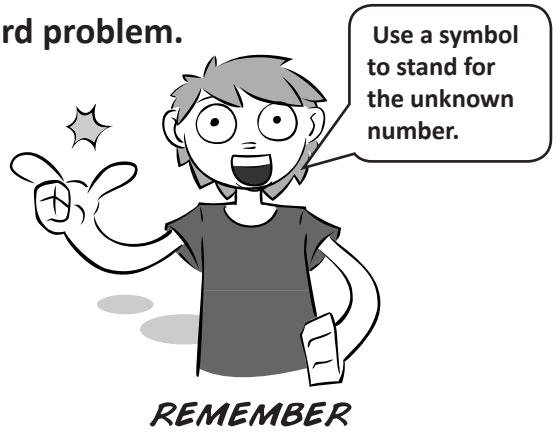
c 50 − ⬡ = 25

⬡ = ☐

d 6 × ♡ = 42

♡ = ☐

2 **Choose an equation from above and write a word problem.**



Equations and equivalence – writing equations for word problems

3 **Write an equation for these word problems. Write an equation using a △ for the unknown number.**

a Mia did 6 push ups every day for 7 days. How many push ups did she do altogether?

△ = ☐

b Josh saved $5 of his pocket money over 8 weeks. How much did Josh save at the end of 8 weeks?

△ = ☐

c There are 28 children in the class. 14 children have brown hair. How many children do not have brown hair?

△ = ☐

d Max has $15 more than I do. If I have $50, how much does Max have?

△ = ☐

4 **If the star is worth the same, what is it worth in this equation?**

☆ + ☆ + ☆ = 36

☆ = ☐

Lolly weigh in

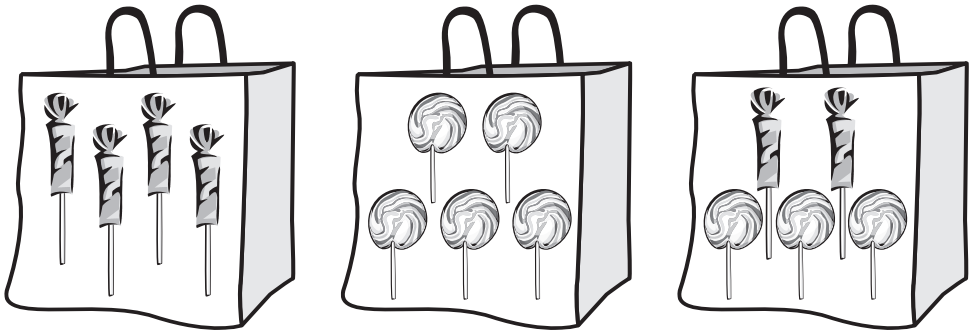
solve

What to do

Work your way through these problems.

Work out what each lolly bag weighs:

a



200 g

250 g

_____ g

b

280 g

320 g

_____ g

c

600 g

480 g

_____ g

Symbol sums — solve

Work out the value of each symbol. If the symbol is repeated it is the same number.

☆ + ☆ = 10

♡ − ☆ = 30

♡ − ☺ = ☆

☆ = ☐

♡ = ☐

☺ = ☐

☆ + ☆ = 14

☺ − ♡ = ☆

♡ − 23 = ☆

☆ = ☐

☺ = ☐

♡ = ☐

☆ + ☆ + ☆ = 21

☆ + ♡ = ☺

♡ + 2 = ☆

☆ = ☐

☺ = ☐

♡ = ☐

☆ × ☆ = 36

♡ − 6 = ☆

☺ × ☆ = ♡

☆ = ☐

☺ = ☐

♡ = ☐